LEADING ROLE OF GRAVITY IN THE STRUCTURE OF SPINNING PARTICLE

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Abstract

The Kerr-Newman solution has g=2 as that of the Dirac electron and is considered as a model of spinning particle in general relativity. The Kerr geometry changes cardinally our representations on the role of gravity in the particle physics. We show that the Kerr gravitational field has a stringy local action and a topological peculiarity which are extended up to the Compton distances, and also a strong non-local action playing the key role in the mass-renormalization and regularization of singularities. The Kerr-Newman gravity determines the structure of spinning particle in the form of a relativistically rotating disk, a highly oblate bag of the Compton radius. Interior of this bag consists of an AdS or dS "false vacuum", depending on the correlation of the mass density and charge. In the same time, the local action of gravitational field may be considered as negligible for regularized particle.

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1 Introduction

The used in QED mass renormalization is universally recognized due to an incredible exactness of its predictions. Although its origin lies in the classical theory of a pointlike electron, there are serious problems with physical interpretation and mathematical correctness of this procedure.

In this paper we consider gravitational field of spinning particle - the Kerr-Newman solution of the Einstein-Maxwell theory. This solution has double gyromagnetic ratio, as that of the Dirac electron and may be considered as a model of electron in general relativity [1, 2, 3, 4, 5, 7]. We show that the mass renormalization and regularization of the singularities in the Kerr-Newman source are realized by gravitational field in a very natural manner. It allows one to conjecture that the methodological problems of QED may be connected with the ignorance of gravity. QED ignores gravitational field arguing that its local action is negligible on the scale of elementary particles. It is true, but only partially. The unusual structure of the Kerr solution gives us a contr-example to this assertion. Taking the parameters of the Kerr-Newman source, charge e, mass m and spin J equal to parameters of elementary particles, one obtains that the Kerr parameter $a = J/m$, which characterizes the radius of the Kerr singular ring, satisfies condition $a >> m$, when the Kerr's event horizon disappears, and the source represents a *naked singular ring of the Compton radius* $a \sim \frac{\hbar}{m}$. We show that the Kerr gravity displays in the Compton the strong local field having a stringy structure, a nontrivial topological peculiarity and has a strong non-local action playing the key role in the mass- renormalization. All these effects must not be ignored in a consistent theory.

1.1 Local and topological peculiarities of the Kerr gravity on the Compton distances

Let us first recall some peculiarities of the Kerr geometry. The local action of the Kerr gravity extends on the Compton distances due to stringy structure of the Kerr source. It was shown [8, 9] that the ringlike Kerr singularity is indeed a string resembling the heterotic string of superstring theory. It breaks drastically all the former arguments based on the assumptions on the spherical symmetry of gravitational field. Indeed, the changes are still more serious, since this singular ring is a branch line of the Kerr space on two

sheets, "positive"($r > 0$) and "negative" ($r < 0$), where r is an oblate radial coordinate of the Kerr oblate coordinate system. Therefore, the Kerr space has twofold topology just in this Compton region. So, this string is "Alice" one [10], and the Kerr ring represents a "mirror gates" in the "Alice mirror world" where the mass and charge have another signs and fields have different directions. It means that the behavior of the particles and fields has to suffer from essential influence in the vicinity of this Compton region. Note, that it is just the region which identified in QED as a region of virtual photons.

Another very important effect of gravity is related to its non-local action. In the next section we show that gravity provides the mass renormalization in a very natural manner. A smooth regularization of the Kerr-Newman solution is considered, leading to a source in the form of a rotating bag filled by a false vacuum. It is shown that gravity controls the phase transition to AdS or dS false vacuum state inside the bag, providing the mass balance.

2 Renormalization by gravity.

Mass of an isolated source is determined by asymptotic gravitational field only, and therefore, it depends only on the mass parameter m which survives in the asymptotic expansion for metric. On the other hand, the total mass can be calculated as a volume integral which takes into account densities of the electromagnetic energy ρ_{em}, material ("mechanical" mass) sources ρ_m and energy of gravitational field ρ_g. The last term is not taken into account in QED, but namely this term provides renormalization. For a spherically symmetric system, the expression may be reduced to an integral over radial distance r

$$m = 4\pi \int_0^\infty \rho_{em} dr + 4\pi \int_0^\infty \rho_m dr + 4\pi \int_0^\infty \rho_g dr. \tag{1}$$

It looks like the expressions in a flat space-time. However, in the Kerr-Schild background it is consequence of the exact Tolman relations taking into account energy of matter, energy of gravitational field (including the contribution from pressure) and rotation [11]. In the well known classical model of electron as a charged sphere with electromagnetic radius $r_e = \frac{e^2}{2m}$, integration in (1) is performed in the diapason $[r_0, \infty]$, where $r_0 = r_e$. The total mass is determined by electromagnetic contribution only, and contribution from gravity turns out to be zero. However, if $r_0 < r_e$, electromagnetic

contribution exceeds the total mass and this exceeding is to compensated by the negative gravitational contribution. Indeed, the results will not depend on the cut parameter r_0 and, moreover, on radial distribution of matter at all. Some of the terms may be divergent, but the total result will not be changed, since divergences will always be compensated by contribution from gravitational term. It shows that, due to the strong non-local action, gravity has to be very essential for elementary particles, on the distances which are very far from the considered usually Planck scale.

2.1 Some more on the structure of the Kerr geometry.

The Schwarzschild's singular point turns in the Kerr solution into a singular ring of the radius $a = J/m$. For $J \sim \hbar$, it is the Compton radius which exceeds for electron the Schwarzschild one at $\sim 10^{22}$ times. Angular momentum $J = \hbar/2$ for parameters of electron is very high, $a >> m$, so the black hole horizons disappear and the source of the Kerr spinning particle represents a naked singular ring which may have some stringy excitations generating the spin and mass of the extended particle - "microgeon" model [3]. The Kerr singular ring cannot be localized inside the region which is smaller of the Compton size, which is similar to the properties of the Dirac electron and to the assertions of QED. One more remarkable structure of the Kerr geometry is PNC (principal null congruence). It is a very important object, since the tangent to this congruence vector k^μ determines the Kerr-Schild ansatz for the metric

$$g^{\mu\nu} = \eta^{\mu\nu} + 2Hk^\mu k^n \qquad (2)$$

(where $\eta^{\mu\nu}$ is the auxiliary Minkowski metric) and also the form of vector potential $A_\mu = \mathcal{A}(x)k_\mu$ for electrically charged solution, i.e. it determines polarization of the gravitational and electromagnetic fields around the Kerr source. For the nonstationary solutions containing the wave excitations [13], PNC determines the directions of radiation which propagates along k^μ.

The Kerr congruence is determined by the Kerr theorem in terms of twistors [14, 15] which form a vortex of lightlike rays, see Fig. 1. One can see, that the Kerr congruence is "in" -going on the "negative" sheet of the Kerr geometry, it passes through the disk spanned by the Kerr ring, and turns out to be "out"-going on the "positive" sheet of the space.

Note, that twistor may be considered as a pair $\{x^\mu, k^/m\}$ which connect a

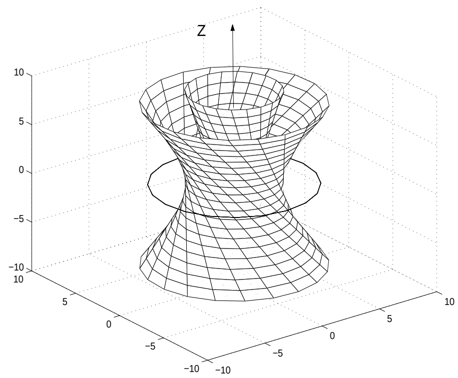

Figure 1: The Kerr singular ring and 3-D section of the Kerr principal null congruence (PNC). Singular ring is a branch line of space, and PNC propagates from "negative" sheet of the Kerr space to "positive " one, covering the space-time twice.

lightlike direction, determined by the null vector k^μ, with a space-time point x^μ, forming a null ray passing via a given point. Instead of the null vector k^μ, the Kerr theorem treats the projective spinor coordinate $Y = \psi^0/\psi^1$ which allows to restore the null vector.

Twistorial structure of PNC forms a vortex *web* which covers the whole space-time, but it focuses on the Kerr singular ring of Compton radius. We will argue in the next section that PNC is a flow of virtual photons (zero point field) which are fall on the particle, excite it, and leave it out-going to infinity.

2.2 Regularization of the Kerr singularity.

When considering the Kerr-Newman solution as a model of electron, one can also assume some other elementary particles. The Kerr-Schild form of metric allows one to consider a broad class of regularized solutions which remove the Kerr singular ring, covering it by a matter source. Usually, the regularized solutions have to retain the Kerr-Schild form of metric and the form of Kerr principal null congruence $k_\mu(x)$, as well as its property to be geodesic and shear-free. The space part $\vec{n}$ of the Kerr congruence $k_\mu = (1, \vec{n})$ has the form of a spinning hedgehog. Indeed, by setting the parameter of rotation a equal to zero, the Kerr singular ring shrinks to singular point,

and $\vec{n}$ takes the usual hedgehog form which is used as an ansatz for the solitonic models of elementary particles and quarks. It suggests that the Kerr spinning particle may have relation not only to electron, but also to the other elementary particles. Indeed, the Kerr-Schild class of metric has a remarkable property, allowing us to consider a broad class of the charged and uncharged, the spinning and spinless solutions from an unified point of view.

The *smooth* regularized sources for the rotating and non-rotating solutions of the Kerr-Schild class have the Kerr-Schild form of metric (2), where the scalar function H has the general form [5, 11]

$$H = f(r)/(r^2 + a^2 \cos^2\theta). \tag{3}$$

For the Kerr-Newman solution function $f(r)$ has the form

$$f(r) \equiv f_{KN} = mr - e^2/2. \tag{4}$$

Regularized solutions have tree regions:

i) the Kerr-Newman exterior, $r > r_0$, where $f(r) = f_{KN}$,

ii) interior $r < r_0 - \delta$, where $f(r) = f_{int}$ and function $f_{int} = \alpha r^n$, and $n \geq 4$ to suppress the singularity at $r = 0$, and provide the smoothness of the metric up to the second derivatives.

iii) a narrow intermediate region $r \in [r_0 - \delta, r_0]$ which allows one to get a smooth solution interpolating between regions i) and ii).

It is advisable to consider first the non-rotating cases, since the rotation can later be taken into account by an easy trick. In this case, taking $n = 4$ and the parameter $\alpha = 8\pi\Lambda/6$, one obtains for the source (interior) a space-time of constant curvature $R = -24\alpha$ which is generated by a source with energy density $\rho = \frac{1}{4\pi}(f'r - f)/\Sigma^2$,

and tangential and radial pressures $p_{rad} = -\rho, \quad p_{tan} = \rho - \frac{1}{8\pi}f''/\Sigma$,

where $\Sigma = r^2$. It yields for the interior the stress-energy tensor in a diagonal form

$T_{\mu\nu} = \frac{3\alpha}{4\pi}diag(1, -1, -1, -1)$, or

$$\rho = -p_{rad} = -p_{tan} = \frac{3\alpha}{4\pi}, \tag{5}$$

which generates a de Sitter interior for $\alpha > 0$, anti de Sitter interior for $\alpha < 0$. If $\alpha = 0$, we have a flat interior which corresponds to some previous

classical models of electron, in particular, to the Dirac model of a charged sphere and to the Lopez model in the form of a rotating elliptic shell [4].

The resulting sources may be considered as the bags filled by a special matter with positive ($\alpha > 0$) or negative ($\alpha < 0$) energy density. The transfer from the external electro-vacuum solution to the internal region (source) may be considered as a phase transition from 'true' to 'false' vacuum in a supersymmetric $U(1) \times \tilde{U}(1)$ Higgs model [5] based on the Witten model for superconducting strings [6]. Assuming that transition region iii) is very thin, one can consider the following graphical representation which turns out to be very useful.

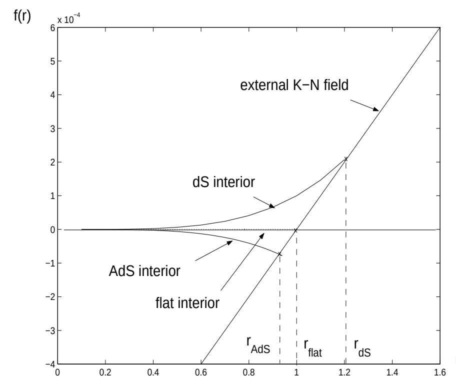

Figure 2: Regularization of the Kerr spinning particle by matching the external field with dS, flat or AdS interior.

The point of phase transition r_0 is determined by the equation $f_{int}(r_0) = f_{KN}(r_0)$ which yields $\alpha r_0^4 = m r_0 - e^2/2$. We have $\rho = \frac{3\alpha}{4\pi}$ and obtain the equation

$$m = \frac{e^2}{2r_0} + \frac{4}{3}\pi r_0^3 \rho. \quad (6)$$

In the first term on the right side, one can easily recognize the electromagnetic mass of a charged sphere with radius r_0, $M_{em}(r_0) = \frac{e^2}{2r_0}$, while the second term is the mass of this sphere filled by a material with a homogenous density ρ, $M_m = \frac{4}{3}\pi r_0^3 \rho$. Thus, the point of intersection r_0 acquires a deep physical meaning, providing the energy balance by the mass formation. In particular, for the classical Dirac model of a charged sphere with radius

$r_0 = r_e = \frac{e^2}{2m}$, the balance equation yields the flat internal space with $\rho = 0$. If $r_0 > r_e$, a material mass of positive energy $M_m > 0$ gives a contribution to total mass m. If $r_0 < r_e$, this contribution has to be negative $M_m < 0$, which is accompanied by the formation of an AdS internal space.

2.3 Transfer to the rotating case.

All the above treatment retains valid for the rotating cases, and for the passage to a rotating case, one has only to set $\Sigma = r^2 + a^2 \cos^2 \theta$, and consider r and θ as the oblate spheroidal coordinates [11].

The Kerr-Newman spinning particle with a spin $J = \frac{1}{2}\hbar$, acquires the form of a relativistically rotating disk, so the board of the disk has $v \sim c$ [11]. The corresponding diagonal stress-energy tensor describes in this case the matter of source in a co-rotating with this disk coordinate system. Disk has the form of a highly oblate ellipsoid with the thickness r_0 and the Compton radius $a = \frac{1}{2}\hbar/m$. Interior of the disk represents a "false" vacuum having superconducting properties which are modelled by the Higgs field. It is a physical realization of the "Alice mirror world" [10]. The charges are concentrated on the surface of this disk, at $r = r_0$. Inside the disk the local gravitational field is negligible.

3 Regularization of the zero-point radiation.

The Kerr singular string may acquire electromagnetic wave excitations from interaction with virtual photons [3, 13, 9]. In classical theory these excitations have to lead to a radiation and non-stationarity of solutions. In the Kerr-Schild formalism [2] electromagnetic excitations and radiation are related to some field $\gamma(x)$, and are described by the $\{13\}$-tetrad components of the self-dual tensor

$$\mathcal{F}_{31} = \gamma Z + (AZ)_{,1} \approx \gamma \frac{1}{r} + (extra\ terms). \qquad (7)$$

It leads to an electromagnetic radiation and a flow of energy along the Kerr congruence, k_μ, and consequently, to a nonstationarity of the solutions. Note, that in quantum theory oscillations are stationary and absence of radiation caused by oscillations is postulated.

The treatment of quantum field theory in curved spaces [16] gives a receipt for transfer from quantum fields to the classical Einstein-Maxwell theory. It shows that quantum fields are concentrated in the divergent vacuum zero point field, and by the transfer to the Einstein-Maxwell equations, the stress-energy tensor has to be regularized by subtraction of the quantum vacuum fields [16].

$$T^{(reg)}_{\mu\nu} = T_{\mu\nu} - < 0|T_{\mu\nu}|0 > . \tag{8}$$

Regularization has to satisfy the condition $T^{(reg)\ \mu\nu}{}_{,\mu} = 0$.

It suggests [13, 9, 14] that the part of stress-energy tensor, which is related to electromagnetic radiation propagating along the Kerr PNC, has to be regularized by a special subtraction as a vacuum field, before the substitution into the Einstein-Maxwell field equations. Twofoldedness of the Kerr geometry confirms this point of view, since *the out-going radiation on the "positive" out-sheet of the metric is compensated by an in-going radiation on the "negative" in-sheet*, see Fig. 1. So, physically, there is no reason for the lost of mass by radiation. It shows, that the term $\mathcal{F}_{31} = \gamma \frac{1}{\tilde{r}}$ has to be identified with the vacuum zero-point field. In this case the electromagnetic excitations on the Kerr background may be interpreted as a resonance of the zero-point fluctuations on the (superconducting) source of the Kerr spinning particle [13, 9]. It was shown [13, 7] that such a regularization may be performed and leads to some modified Kerr-Schild equations. Although, the exact nontrivial solutions of the regularized system have not been obtained so far, there were obtained corresponding exact solutions of the Maxwell equations which show that any "aligned" excitation of the Kerr geometry leads to the appearance of extra "axial" singular lines (strings) [13, 12] which are semi-infinite. These strings are related to solutions of the Dirac equation, are modulated by de Broglie periodicity and may be considered as the physical carriers of wave function.

Conclusion. Multi-sheeted twistorial web

The obtained recently multiparticle Kerr-Schild solutions [15] support the above point of view. It was obtained that the Kerr theorem has a wonderful consequence: the twistorial webs related to different particles penetrate trough each other without interaction. The extended twistorial space-time, consisting of the pair $\{x, Y\}$, where $x \in M^4$, $Y \in CP^1$, turns out to be

multi-sheeted, similar to multi-sheeted Riemann holomorphic surfaces. In the same time, the obtained exact multiparticle solutions show, that metric turns out to be singular along some twistor lines which are *common* for twistorial structures of two interacting particles. For any two interacting particles one can find two such common twistor lines of opposite direction. As a result, a pair of semi-infinite axial string appears by interaction with any external particle, and interaction with many surrounding particles results that almost all the twistorial rays of the Kerr PNC turn out to be singular. It may be interpreted as an evidence that PNC describes a virtual multiparticle interaction, and that related to PNC fields have to be regularized as belonging to the virtual vacuum fields. We arrive at the conclusion that the twistorial structure of the Kerr PNC belongs to the vacuum zero-point field and hints at a multi-sheeted twistorial texture of vacuum [13, 7].

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